

สูตร ตรีโกณ (Trigonometry)

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A - \tan^2 A = 1$$

$$\operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\sin A \cdot \operatorname{cosec} A = 1$$

$$\cos A \cdot \sec A = 1$$

$$\tan A \cdot \cot A = 1$$

$$\tan A = \frac{\sin A}{\cos A}$$

ให้ $A + B = 90^\circ$, $\sin A = \cos B$

$$\sec A = \operatorname{cosec} B$$

$$\tan A = \cot B$$

มุมติดลบ $\sin(-A) = -\sin A$

$$\cos(-A) = \cos A$$

$$\tan(-A) = -\tan A$$

ผลบวก - ผลต่างของมุม

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

มุมสองเท่า $\sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

มุมสามเท่า $\sin 3A = 3 \sin A - 4 \sin^3 A$

$$\cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

มุมครึ่งเท่า $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$= \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$$

ผลคูณ → ผลบวก

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

ผลบวก → ผลคูณ

$$\sin A + \sin B = 2 \sin \frac{(A+B)}{2} \cos \frac{(A-B)}{2}$$

$$\sin A - \sin B = 2 \cos \frac{(A+B)}{2} \sin \frac{(A-B)}{2}$$

$$\cos A + \cos B = 2 \cos \frac{(A+B)}{2} \cos \frac{(A-B)}{2}$$

$$\cos A - \cos B = -2 \sin \frac{(A+B)}{2} \sin \frac{(A-B)}{2}$$

สูตรตรีโกณ(ต่อ)

ผลต่างกำลังสอง

$$\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

$$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$$

ค่าต่ำสุดหรือสูงสุดของฟังก์ชัน

$$y = a \sin \theta + b \cos \theta \text{ คือ } \pm \sqrt{a^2 + b^2}$$

กฎของไซน์ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

กฎของโคไซน์

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = b^2 + a^2 - 2ab \cos C$$

กฎของโปรเจกชัน

$$a = b \cos C + c \cos B$$

$$b = a \cos C + c \cos A$$

$$c = a \cos B + b \cos A$$

พื้นที่สามเหลี่ยม

$$\Delta ABC = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

ให้มุม $A + B + C = 180^\circ$

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$$

$$\sin 54^\circ = \frac{\sqrt{5}+1}{4} = \cos 36^\circ$$

เอกลักษณ์ ของ Arc

$$\arcsin x + \arccos x = \frac{\pi}{2}$$

$$\arctan x + \operatorname{arccot} x = \frac{\pi}{2}$$

$$\operatorname{arccosec} x + \operatorname{arcsec} x = \frac{\pi}{2}$$

$$\operatorname{arctan} x + \operatorname{arctan} y = \operatorname{arctan} \left(\frac{x+y}{1-xy} \right)$$

$$\arcsin x = \arccos \sqrt{1-x^2}$$

$$\arccos x = \arcsin \sqrt{1-x^2}$$

$$2 \operatorname{arctan} x = \operatorname{arctan} \left(\frac{2x}{1-x^2} \right)$$

$$\arcsin(\sin x) = x$$

$$\sin(\arcsin x) = x$$

$$\arccos(\cos x) = x$$

$$\cos(\arccos x) = x$$

$$\operatorname{arctan}(\tan x) = x$$

$$\tan(\operatorname{arctan} x) = x$$

Function	โดเมน	เรนจ์
Y = arcsin x	[-1,1]	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
Y = arccos x	[-1,1]	$[0, \pi]$
Y = arctan x	R	$(-\frac{\pi}{2}, \frac{\pi}{2})$
Y = arcsec x	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$
Y = arccot x	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
Y = arccot x	R	$(0, \pi)$

$$\sin 72^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \cos 18^\circ$$

$$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ$$

สูตร ตรีโกณ (Trigonometry)

ผลบวก - ผลต่างของมุม

1. $\sin (A+B) = \dots\dots\dots$
2. $\sin (A-B) = \dots\dots\dots$
3. $\cos (A+B) = \dots\dots\dots$
4. $\cos (A-B) = \dots\dots\dots$
5. $\tan (A+B) = \dots\dots\dots$
6. $\tan (A-B) = \dots\dots\dots$
7. $\sin 2A = \dots\dots\dots$
8. $\sin 2A = \dots\dots\dots$
9. $\cos 2A = \dots\dots\dots$
10. $\cos 2A = \dots\dots\dots$
11. $\cos 2A = \dots\dots\dots$
12. $\sin 3A = \dots\dots\dots$
13. $\cos 3A = \dots\dots\dots$
14. $\tan 3A = \dots\dots\dots$
19. $\text{Cot } 3A = \dots\dots\dots$
20. $\sin \frac{A}{2} = \dots\dots\dots$
21. $\cos \frac{A}{2} = \dots\dots\dots$

22. $\tan \frac{A}{2} = \dots\dots\dots$
23. $2\sin A \cos B = \dots\dots\dots$
24. $2\cos A \sin B = \dots\dots\dots$
25. $2\cos A \cos B = \dots\dots\dots$
26. $2\sin A \sin B = \dots\dots\dots$
27. $\sin A + \sin B = \dots\dots\dots$
28. $\sin A - \sin B = \dots\dots\dots$
29. $\cos A + \cos B = \dots\dots\dots$
30. $\cos A - \cos B = \dots\dots\dots$

สูตรตรีโกณ(ต่อ)

1. $\sin(A + B)\sin(A - B) = \dots\dots\dots$

2. $\cos(A + B)\cos(A - B) = \dots\dots\dots$

ค่าต่ำสุดหรือสูงสุดของฟังก์ชัน

3. $y = a \sin\theta + b \cos\theta$ คือ $\dots\dots\dots$

4. กฎของไซน์ $\dots\dots\dots = \dots\dots\dots = \dots\dots\dots$

5. กฎของโคไซน์

$a^2 = \dots\dots\dots$

$b^2 = \dots\dots\dots$

$c^2 = \dots\dots\dots$

6. กฎของโปรเจกชัน

$a = \dots\dots\dots$

$b = \dots\dots\dots$

$c = \dots\dots\dots$

7. พื้นที่สามเหลี่ยม

$\Delta ABC = \dots\dots\dots = \dots\dots\dots = \dots\dots\dots$

ให้มุม $A + B + C = 180^\circ$

8. $\sin A + \sin B + \sin C = \dots\dots\dots$

9. $\cos A + \cos B + \cos C = \dots\dots\dots$

10. $\tan A + \tan B + \tan C = \dots\dots\dots$

เอกลักษณ์ ของ Arc

11. $\arcsin x + \arccos x = -$

12. $\arctan x + \text{arccot } x = -$

13. $\sin 18^\circ = \dots\dots\dots = \dots\dots\dots$

14. $\sin 72^\circ = \dots\dots\dots = \dots\dots\dots$

15. $\sin 54^\circ = \dots\dots\dots = \dots\dots\dots$

16. $\sin 36^\circ = \dots\dots\dots = \dots\dots\dots$

17. $\text{arccosec } x + \text{arcsec } x = -$

18. $\arctan x + \arctan y = \arctan(\text{—————})$

19. $\arcsin x = \arccos \dots\dots\dots$

20. $\arccos x = \arcsin \dots\dots\dots$

21. $2\arctan x = \arctan(\dots\dots\dots)$

22. $\arcsin(\sin x) =$

23. $\sin(\arcsin x) =$

24. $\arccos(\cos x) =$

25. $\cos(\arccos x) =$

26. $\arctan(\tan x) =$

27. $\tan(\arctan x) =$

Function	โดเมน	เรนจ์
28. $Y = \arcsin x$		
29. $Y = \arccos x$		
30. $Y = \arctan x$		
31. $Y = \text{arccosec } x$		
32. $Y = \text{arcsec } x$		
33. $Y = \text{arccot } x$		